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Limçaon, Polar Curves and Pivoting Figures

Elizabeth F <elizabeth@citibob.net> 8 July 2025 at 06:20
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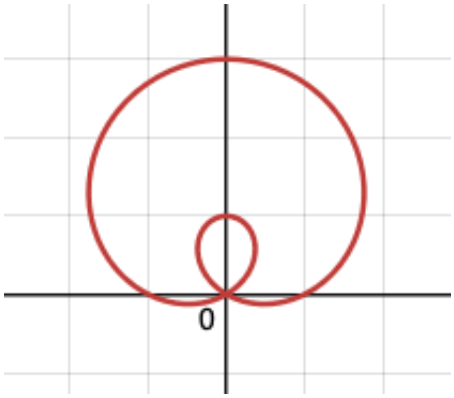
Audrey,

One student brought up a question about Limaçon and other polar curves, such as cardioid, arising from polar coordinates. Please forward this email to him (and others in the program who might be interested)

Thank you,
-- Elizabeth Fischer5

Limçaon

The shape he showed me was r = 1 +2 sin(theta):



At the time I had not thought about them. But upon further reflection... polar curves in general may be a useful way to think about pivoting figures, which are created by anchoring one foot into the ice and using the other skate to create a figure around that origin point. The oldest known ice figure is, in fact, the Pivoting Heart figure described by R. Jones in his book *A Treatise on Skating* in 1780:
https://www.james-gillray.org/pdfs/treatise-skating-jones.pdf

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ſtrokes, you will be able to go at a great rate.

To cut the Figure of a Heart on one Leg.

This is a pleaſing manœuvre, and but lately known; it is difficult, though graceful if well done; the method is as follows: firſt ſet off, with a ſweep on the outſide, on the right leg; and when you think you have formed half the figure of a heart, which you will almoſt naturally do in common rolling, turn yourſelf ſuddenly half round; then throw yourſelf on the inſide edge, and by looking to the right you will move backwards. This motion muſt be continued till you come to the place where you began the heart; it would be rather difficult to deſcribe in what manner the arms ſhould be uſed, nor is it neceſſary, becauſe thoſe who are

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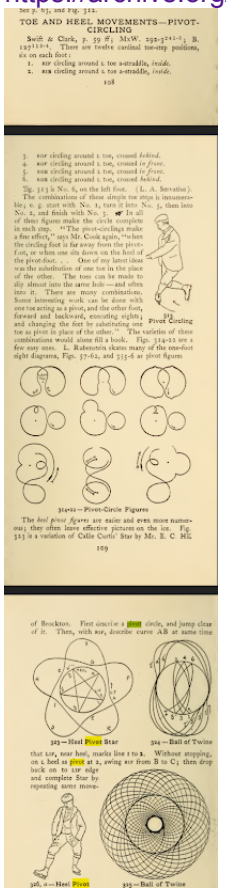
are ſuch proficients as to attempt this manœuvre, will certainly know how to employ them.

There are many other movements performed on skates, beſides thoſe I have treated of; but, as they are neither graceful nor pleaſing, I ſhall here conclude, by ſaying, thoſe who can perform all the manœuvres mentioned in this treatiſe, will have no occaſion for any further inſtructions.

FINIS.

Unfortunately he did not provide us an illustration of that figure. But here is a recent example of it. (Note how he draws out the shape with his heel as a guide before attempting to trace over it with the pivot action): https://www.youtube.com/watch?v=gojEc_orLQ4

Pivot figures in general, pivoting on howe and teel, were well studied and developed here (Browne, 1907: https://archive.org/details/handbookoffigure00brow/page/19/mode/1up?view=theater):


TOE AND HEEL MOVEMENTS—PIVOT-CIRCLING

Pivot Circling

314-22—Pivot-Circle Figures

323—Heel Pivot Star

324—Ball of Twine

325—Ball of Twine

Some of these pivot figures have loops in them. I must admit I do not know how a loop works inside a pivoting figure without getting the body all tangled up. But clearly they had a way to do it, which is really cool. These kinds of pivot loops would be related to polar curves because of the way they are made, essentially, with a human compass. They are different and distinct from the Clothoid loops I talked about, which are made based on physics and smoothness considerations. In essence the Limaçon is "pure" geometry, whereas the Clothoid is physic.s

Pivoting was used in the Panin Lake Special Figure I shared in the slides, and also in the *Tour de Force* ball of twine, which was forgotten knowledge until recently:

https://www.youtube.com/watch?v=9cdW-iCDo8w

Pivoting was also used last October for Shepherd Clark's spiral. Here you can see him finishing up the spiral, with the pivot point in the middle (where he stuck his toe pick in the ice).

All these pivot figures are related to radial coordinates because they involve pivoting the body around and varying the radius by controlling how far apart the legs are held. Here are some more pivoting figures:

https://www.youtube.com/watch?v=Esx6LbhV4-8
https://www.youtube.com/watch?v=BDI4D4FgK7I
https://www.youtube.com/shorts/81jN12EVaBo

The relationship between pivoting figures and well-defined mathematical polar curves is clearly an under-researched topic! Please feel free to reach out to me if you wish to discuss further.