When Pigs Skate:
The Two Most Perfect Shapes

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Presented at The
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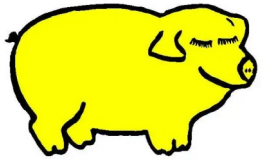
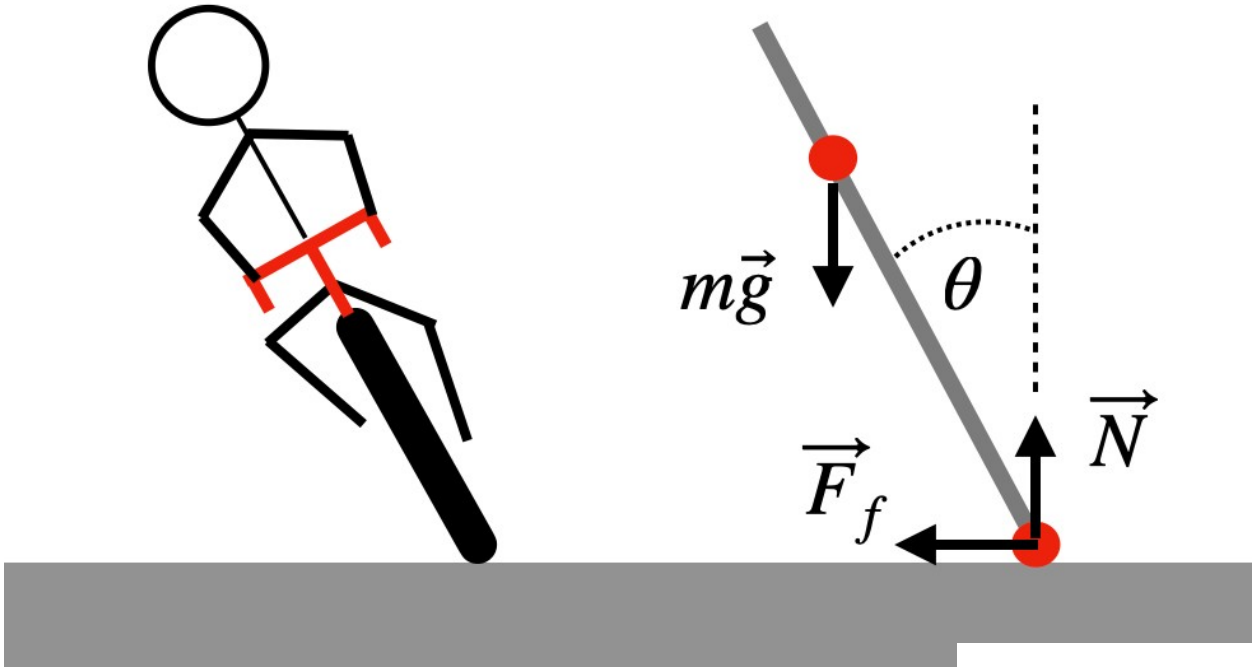


Origins of Figure Skating

Lean and Circles

From USA Cycling

The Lean Equation

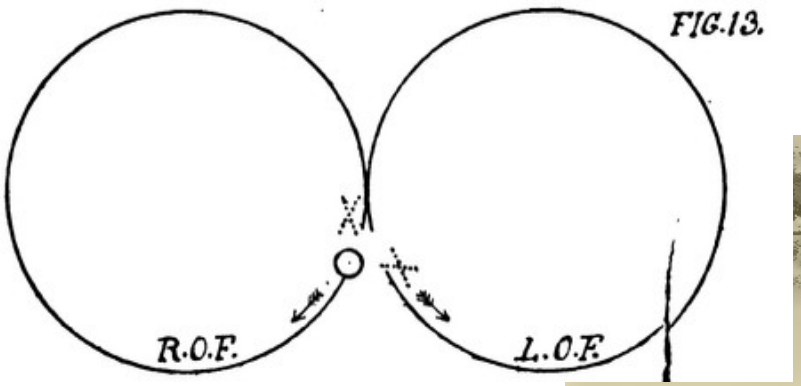


V = speed
r = radius of circle

$$\frac{v^2}{rg} = \theta = \frac{\omega^2}{g}$$

From *Why Do Racing Motorcycles Lean So Much? It's Physics*

Two Circles = Figure 8



Top: From George Meagher,
Figure and Fancy Skating, 1895

Right: From *Skating with Bror Meyer, 1921*

What is a Figure?

- A named pattern of movement
  - Figure Dancing

Figure 1. An 1821 image of a French promenading figure.

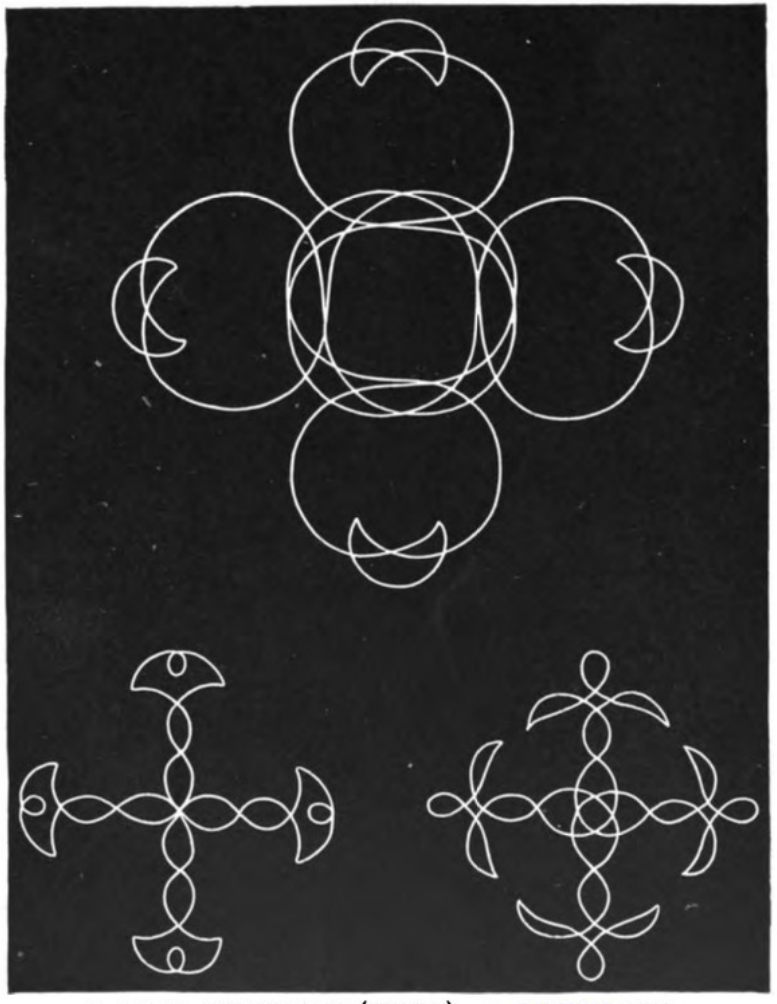
https://www.regencydances.org/paper004.php

What is a Figure?

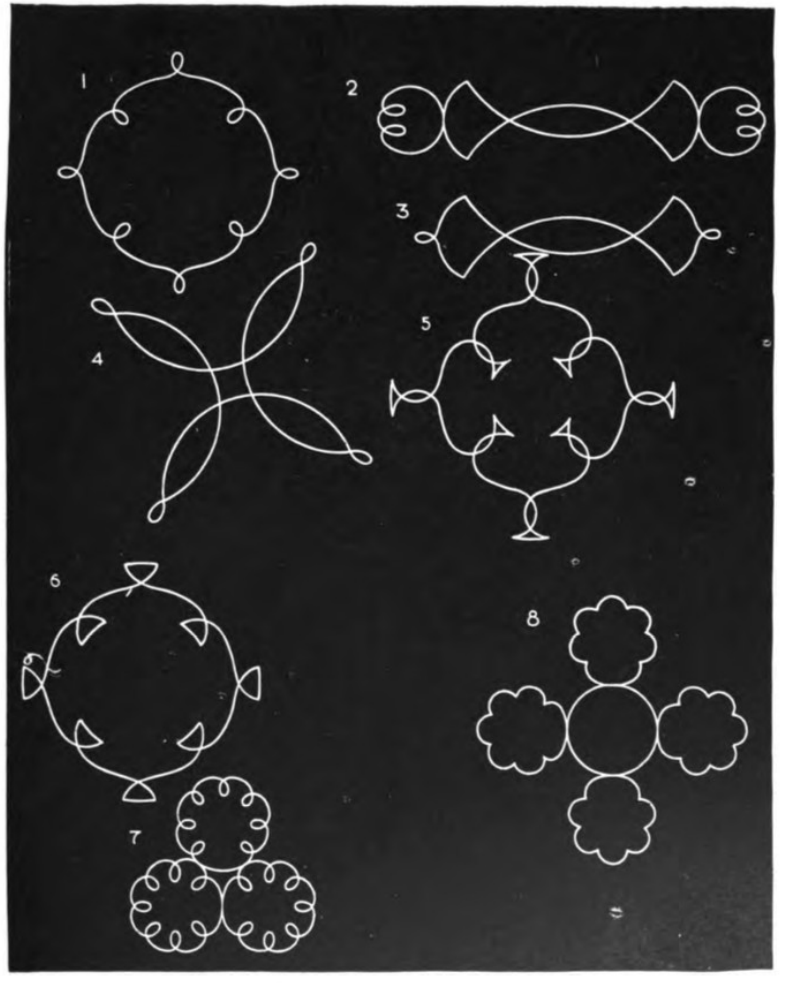
- The imprint left on the ice after performing a figure

Figure-Skating

Special Figures
(George Meagher, 1919)



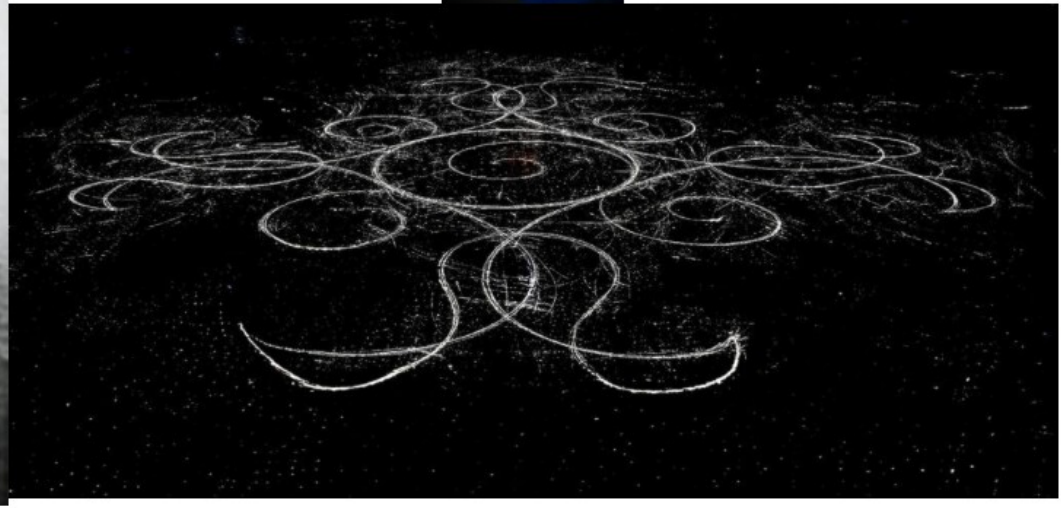
1. BEAKS AND CHANGES (WINZER). 2. SALCHOW STAR.
3. HUGEL STAR.



MEAGHER'S SPECIAL FIGURES.—SET V.

1. Loops. 2. Rockers, loops, and counters. 3. Rockers, loops, and counter.
4. Loops. 5. Crosscuts. 6. Swedish crosscuts. 7. Loops. 8. Threes.

Panin Special Lake Figure, 1908

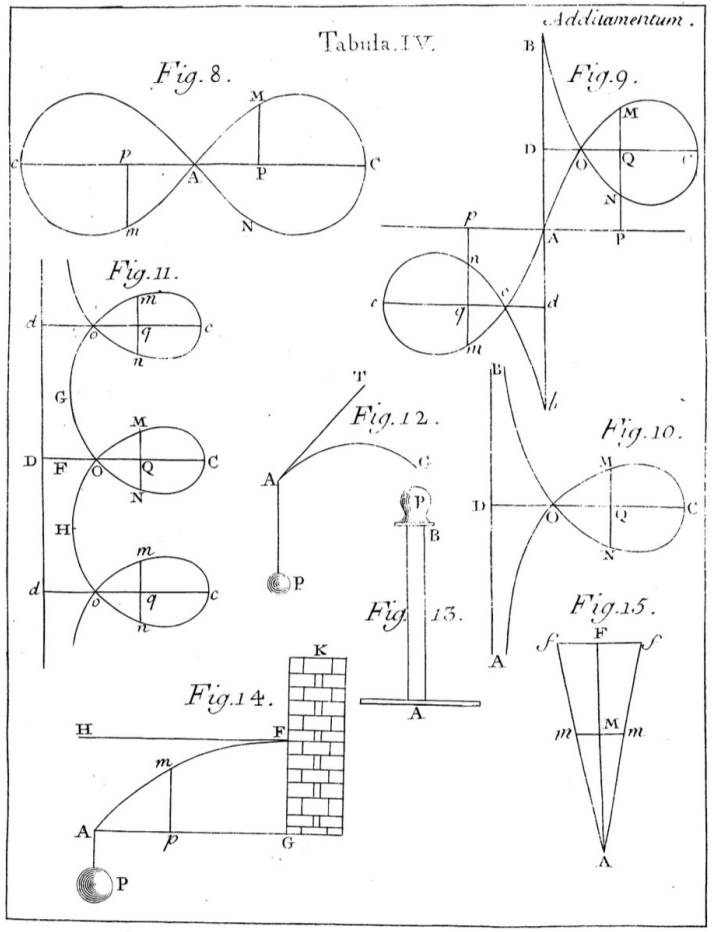


Spiral by Shepherd Clark, 2024. Used by permission of Shepherd Clark.

James Bernoulli, 1691

What shape must a pre-curved wire spring be in such that, when flattened by pressing on the free end, it becomes a straight line?

Leonhard Euler, *elastica*, 1744

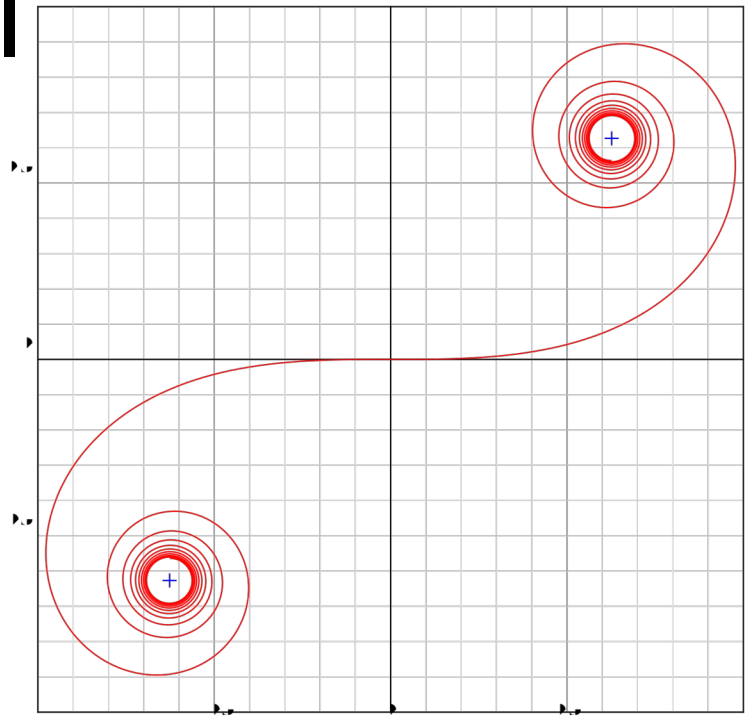


A method for finding curved lines enjoying properties of maximum or minimum, or solution of isoperimetric problems in the broadest accepted sense

(eg: Shapes made when bending elastic things)

Euler Spiral

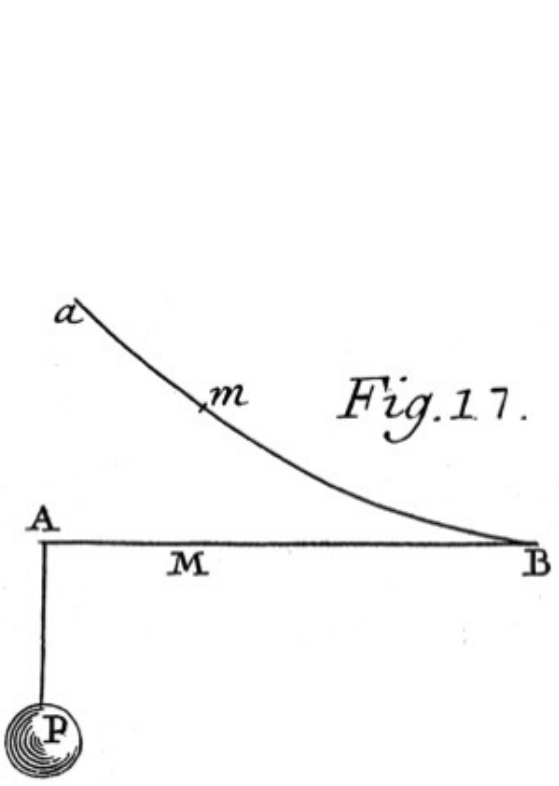
An Euler spiral is a curve whose curvature changes linearly with its curve length (the curvature of a circular curve is equal to the reciprocal of the radius). This curve is also referred to as a clothoid or Cornu spiral.



$$C(L) = \int_0^L \cos\left(s^2\right)\, ds$$

$$S(L) = \int_0^L \sin\left(s^2\right)\, ds$$

From Wikipedia



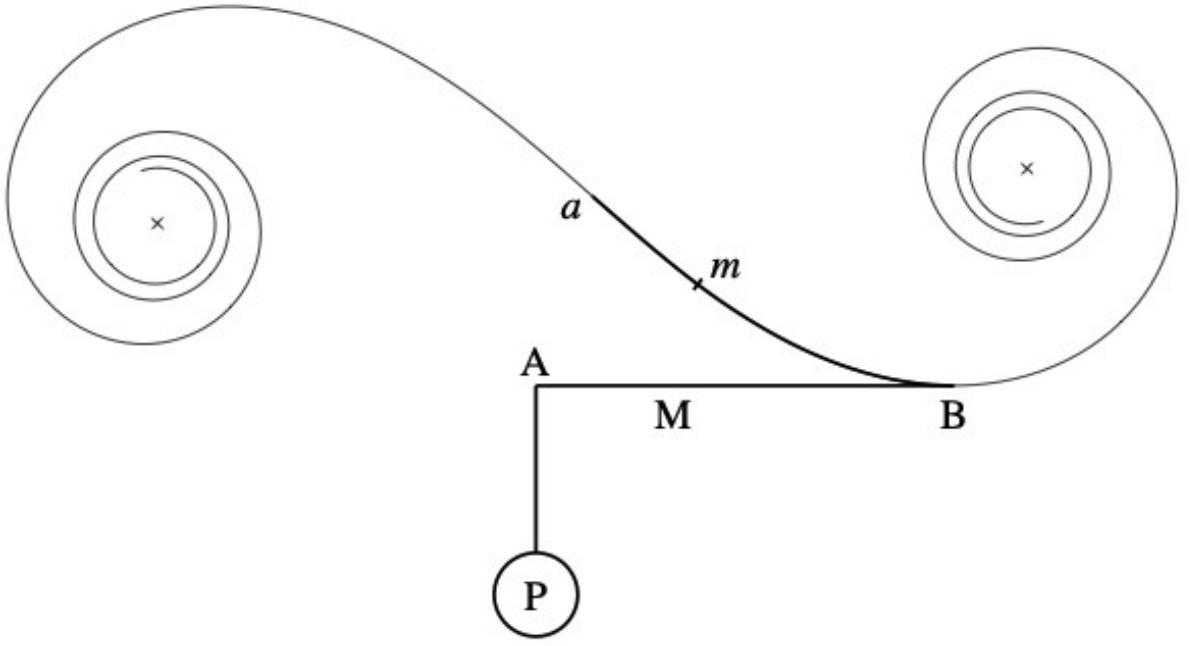


Figure 5: Reconstruction of Euler's Fig. 17, with complete spiral superimposed.

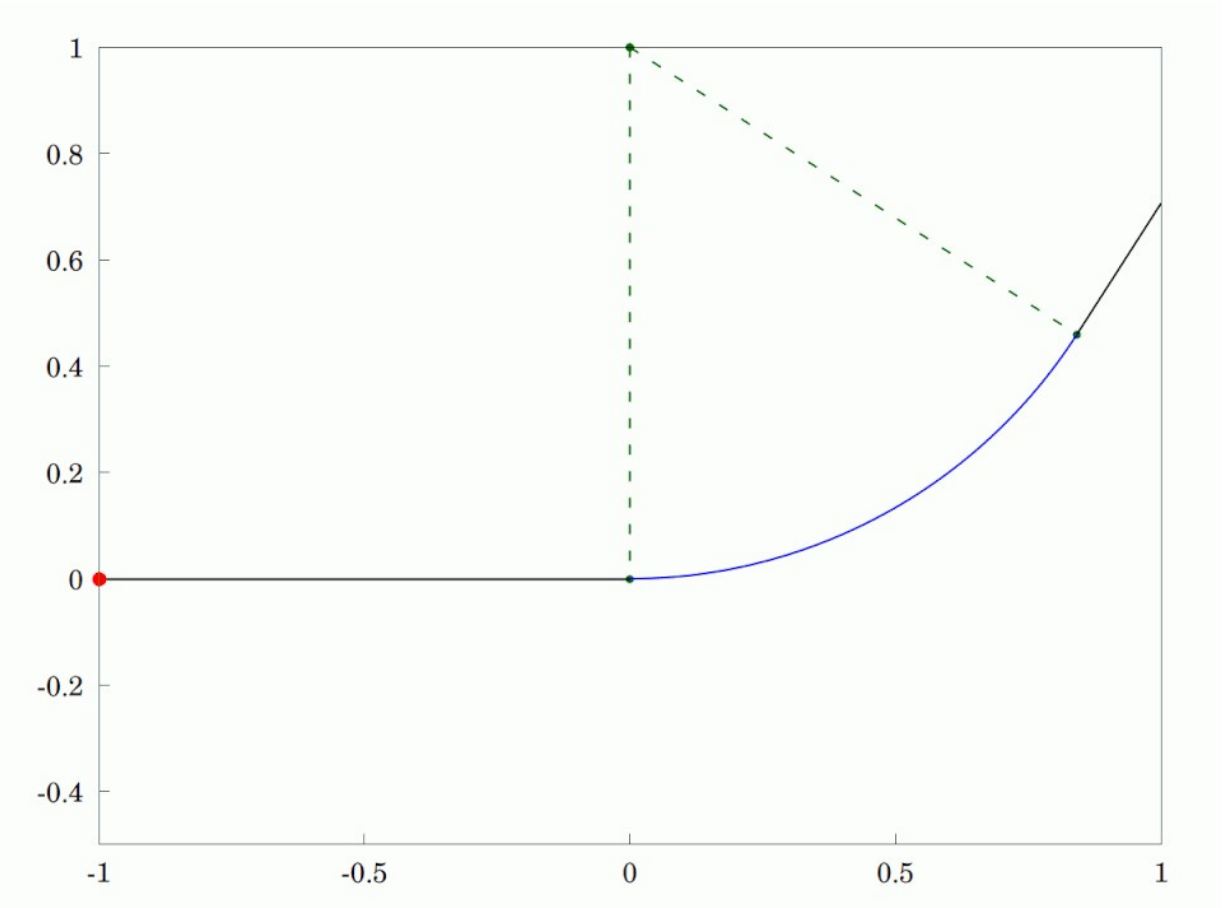
From The Euler spiral: a mathematical history, Raph Levien, Technical Report No. UCB/EECS-2008-111
http://www.eecs.berkeley.edu/Pubs/TechRpts/2008/EECS-2008-111.html

Fresnel, diffraction: 1818

Augustin Fresnel, working in 1818 on the diffraction of light, developed the Fresnel integral that defines the same spiral. He was unaware of Euler's integrals or the connection to the theory of elasticity. In 1874, Alfred Marie Cornu showed that diffraction intensity could be read off a graph of the spiral by squaring the distance between two points on the graph. In his biographical sketch of Cornu, Henri Poincaré praised the advantages of the "spiral of Cornu" over the "unpleasant multitude of hairy integral formulas". Ernesto Cesàro chose to name the same curve "clothoid" after Clotho, one of the three Fates who spin the thread of life in Greek mythology.[2]

(Wikipedia)

Railroad Curve: Circular



Arrow: Centripetal Acceleration (lateral forces felt by passengers)

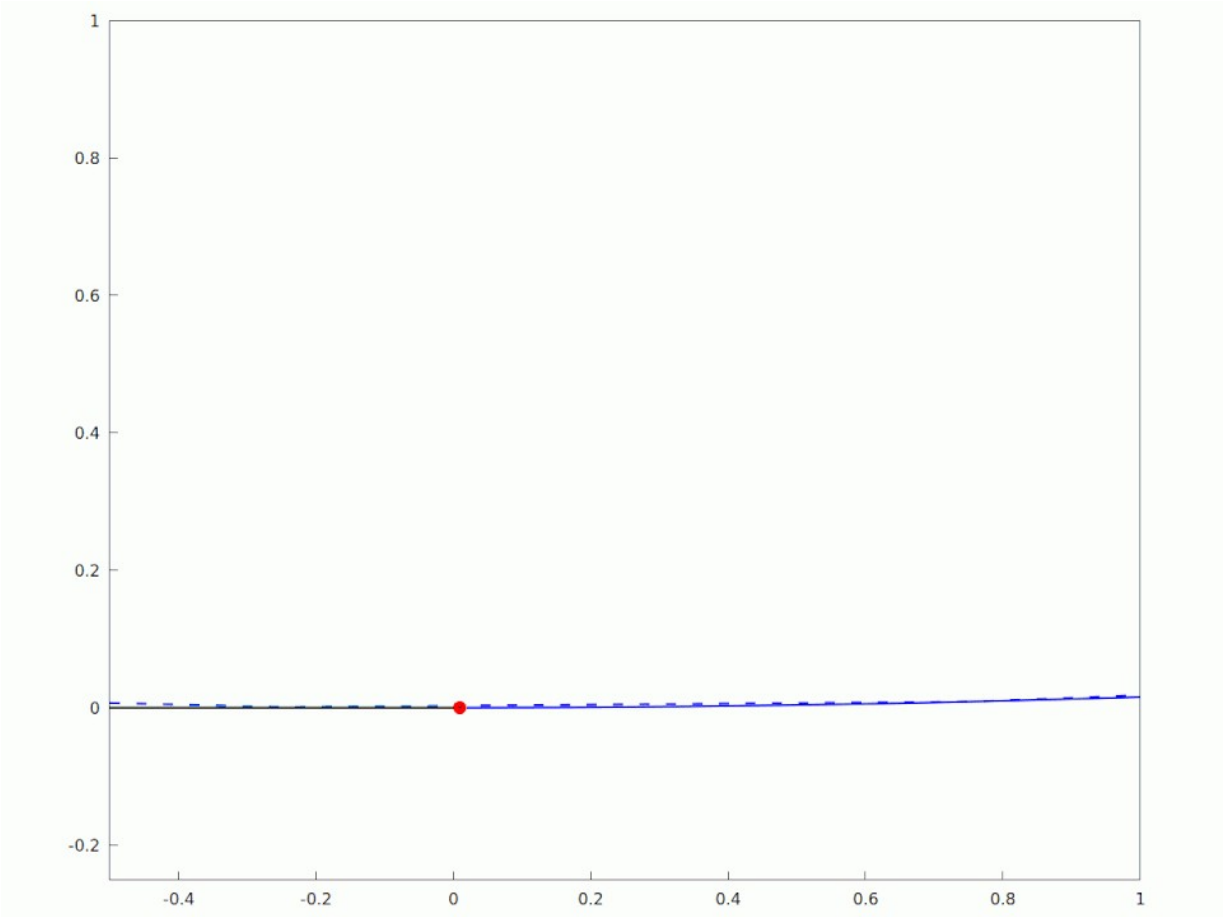
Jerk: Change in acceleration

- Infinite jerk entering and leaving circular arc!
- High G-forces during arc!

See animated GIF on Wikipedia

Track Transition Curve

(Rankine, 1862, Holbrook 1880, Talbot 1890, Glover 1900)



See animated GIF on Wikipedia

Arrow: Centripetal Acceleration (lateral forces felt by passengers)

Jerk: Change in acceleration

- Constant jerk within Euler spiral.
- Lower maximum G-force in spiral.
- Also used on highways

Flip Flap Railway

(Lina Beecher, Coney Island, 1895)

The coaster.. was one of the first looping roller coasters to operate in North America. It was also notable for its engineering as well as the extreme G-force this engineering inflicted on riders...

The circular nature of the coaster's loop, as well as its relatively small diameter of 7.6 metres (25 ft),[3] meant that it could produce forces of approximately 12 g_0 (120 m/s^2).[6] This caused riders to often experience discomfort and neck injuries from whiplash.

G-forces prevented >2 car trains, limited profitability. Closed in 1902.

(from Wikipedia)

Loop the Loop Roller Coaster

(Green & Prescott, Coney Island, 1898)

From Wikipedia

Modern Roller Coasters

(1950's -- present)

Figure 1: Examples of loop shapes. The red loop to the left is from Loopen at Tusenfryd in Norway (Vekoma, Corkscrew, 1988) . The yellow loop in the middle is from HangOver (Vekoma, Invertigo, 1996) at Liseberg, now relocated to Sommerland Syd in Southern Denmark. The loop to the right is from the newly opened "Kanonen" (Launch Coaster, Intamin/Stengel, 2005) at Liseberg [1]. Exercise for the reader: The Kanonen train has a length of 9.5m and takes about 1.3s to pass the top of the loop. Use the photograph to estimate the g-force of the rider in the top of the loop. Does it make any difference whether you sit in the front, back or middle? How much?

https://physics.gu.se/LISEBERG/eng/loop_pe.html

The Euler spiral of ~~rat~~ pig whiskers

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Abstract

This paper reports on an analytical study of the intrinsic shapes of 523 whiskers from 15 rats. We show that the variety of whiskers on a rat's cheek, each of which has different lengths and shapes, can be described by a simple mathematical equation such that each whisker is represented as an interval on the Euler spiral. When

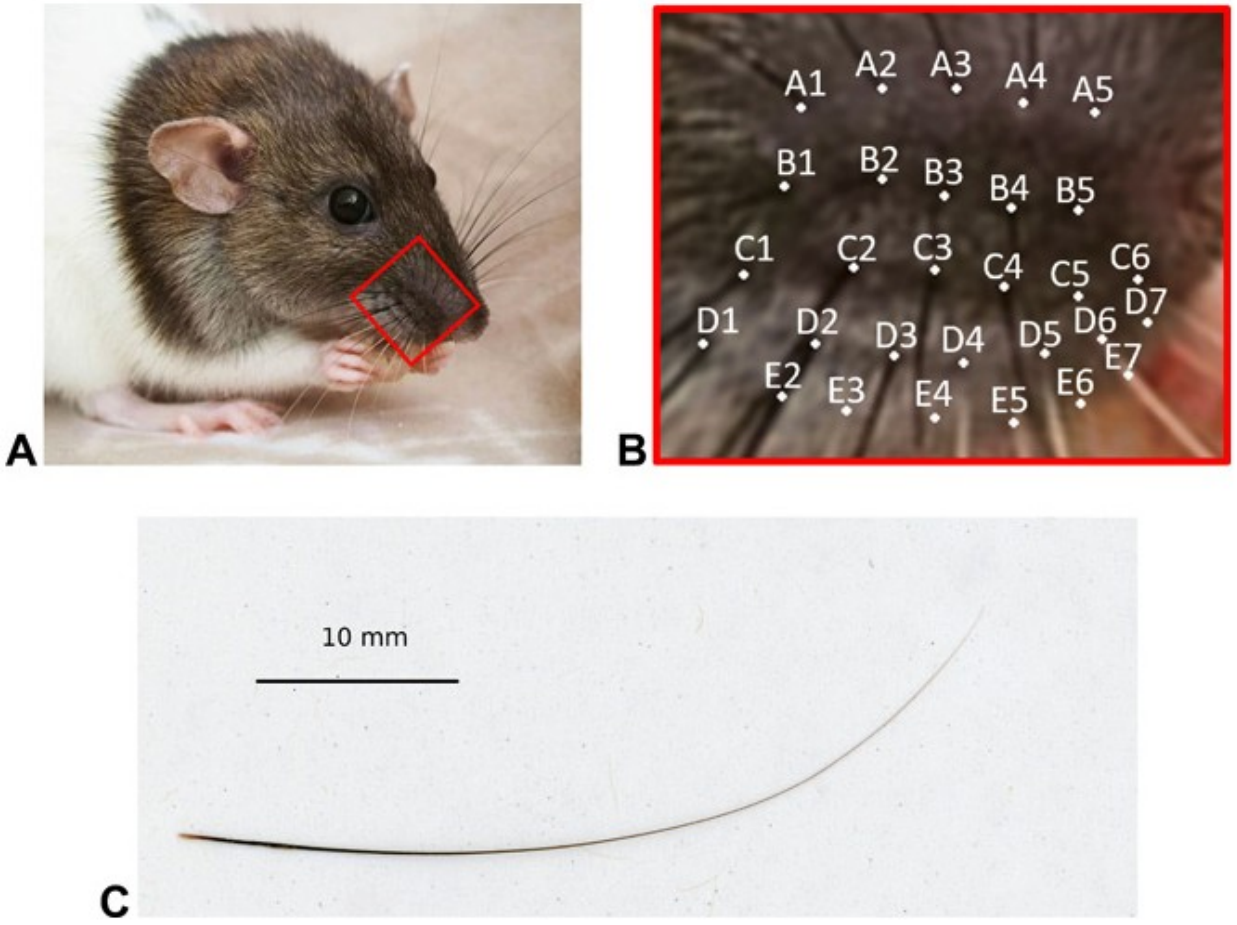


Fig. 1 A rat and its whiskers.

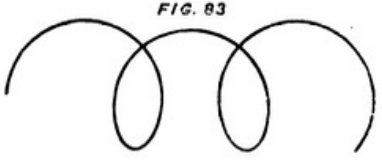
(**A**) A photograph of a rat (photo credit: Maria Panagiotidi, University of Salford). (**B**) The mystacial pad with labeled locations of the base points of 30 whiskers at the right side of a rat. The mystacial pad matrix has five rows (A to E) and seven columns (1 to 7); for five entries, the whiskers are absent. (**C**) A two-dimensional scan of a whisker.

Loop Figure (pre-1849)

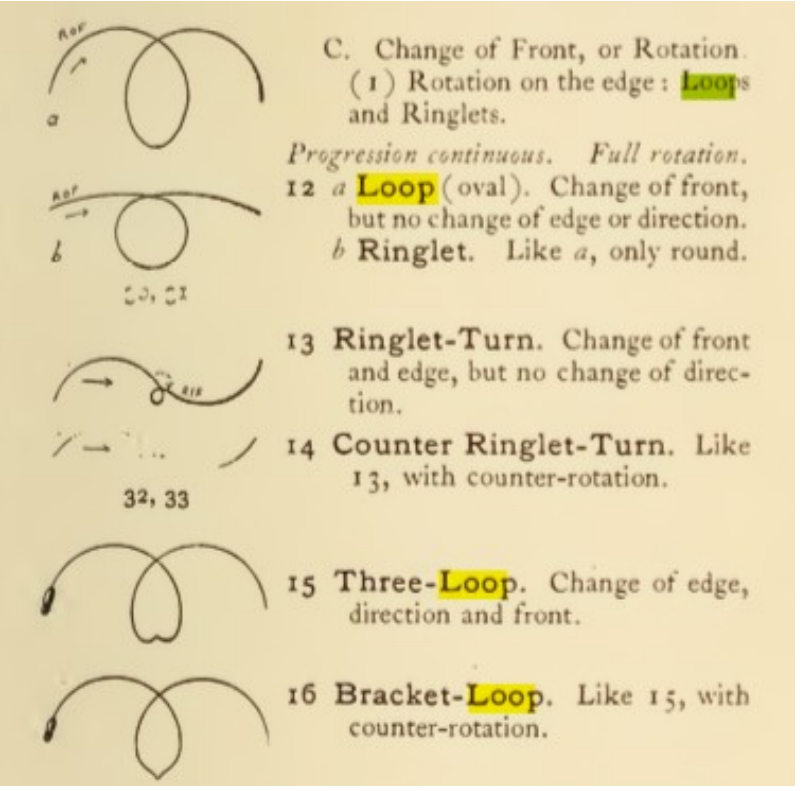
THE SMALL LOOP.

This figure, on a large scale, has been already noticed, and the reason we class it again here is because in its present dwarfed form the skater is no longer able to retain his usual neat attitude: he will have to swing the leg out to the front to give himself the desired twirl or spin at the right moment, and he may, perhaps, require skates rather more curved. Therefore this figure is rarely practised in England. It is a very old one, and was skated by one of the writers twenty years ago. It can be done on all the edges A, B, C, D, and doubled thus (Fig. 83).

FIG. 83



Vandervell and Witham, A System of Figure-Skating, London, 1869



C. Change of Front, or Rotation.
(1) Rotation on the edge: Loops and Ringlets.

Progression continuous. Full rotation.

12 *a* **Loop** (oval). Change of front, but no change of edge or direction.
b **Ringlet.** Like *a*, only round.

30, 31

13 **Ringlet-Turn.** Change of front and edge, but no change of direction.

14 **Counter Ringlet-Turn.** Like 13, with counter-rotation.

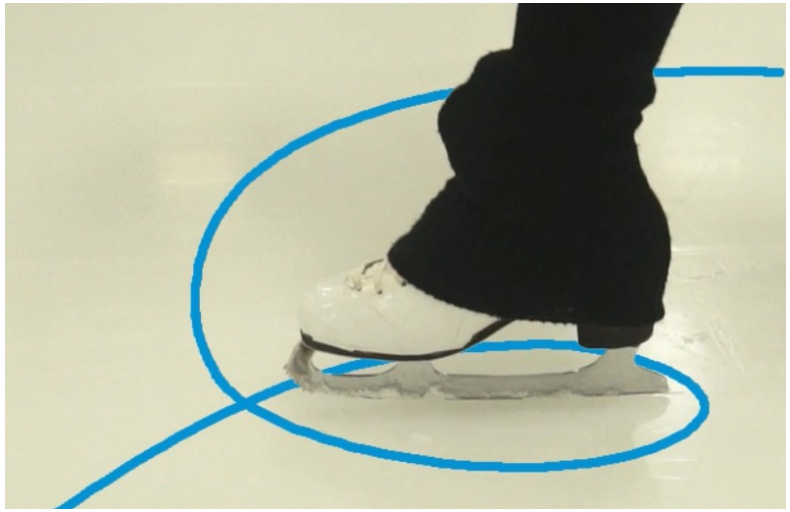
32, 33

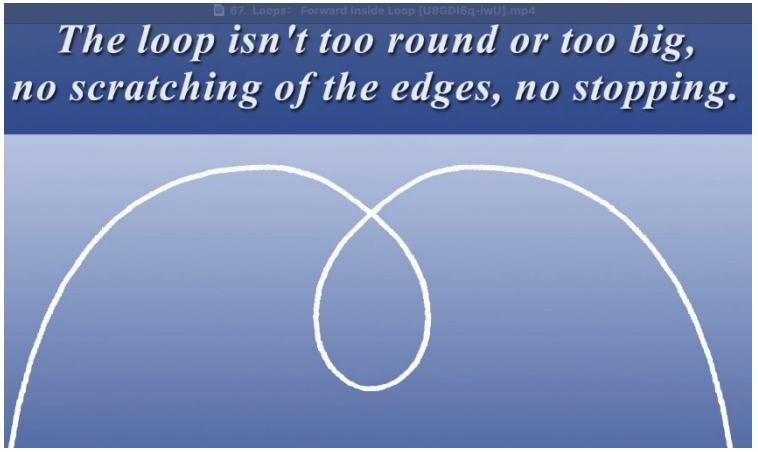
15 **Three-Loop.** Change of edge, direction and front.

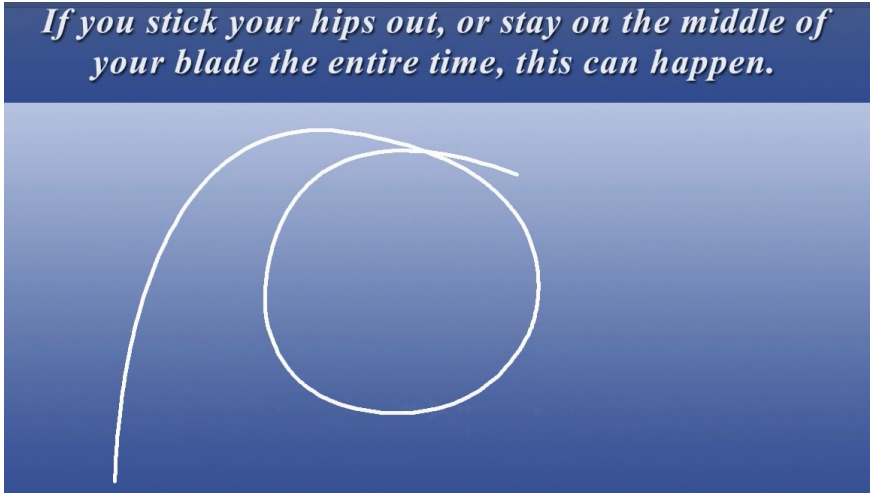
16 **Bracket-Loop.** Like 15, with counter-rotation.

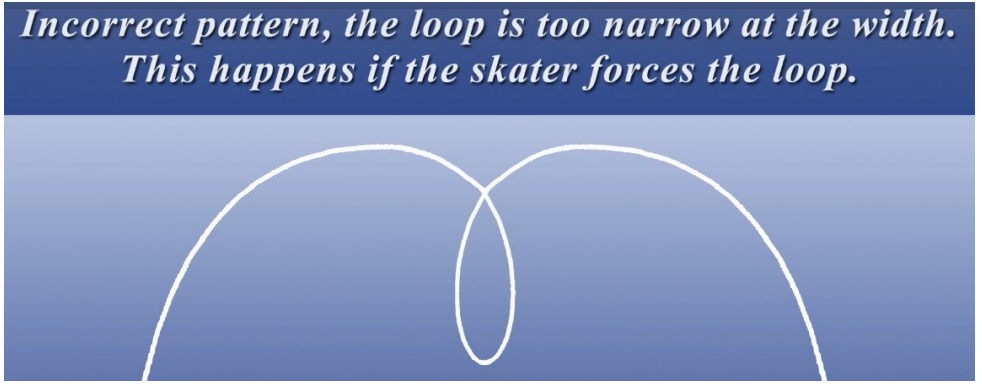
Browne, A Handbook of Figure Skating, Springfield, MA, 1907

How do Skaters Talk about Loop Shape?



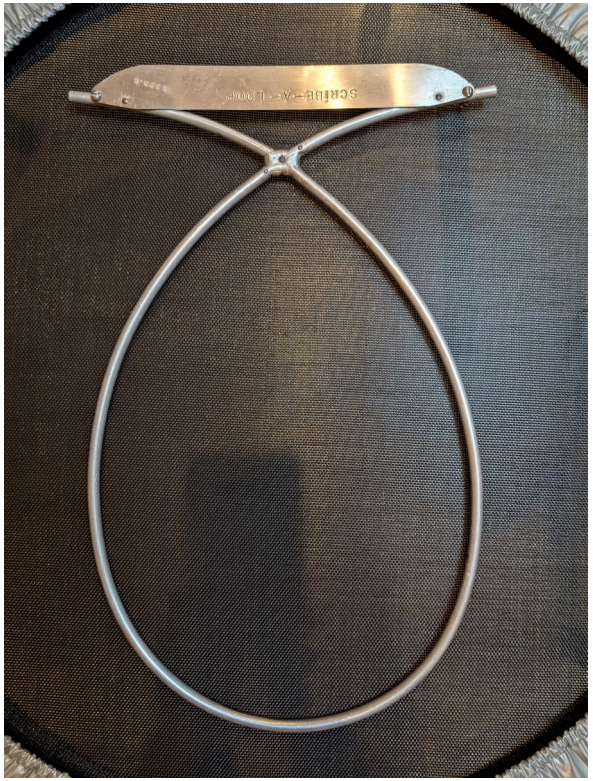






Kseniya and Oleg, Forward Inside Loops, YouTube

Scribe-a-Loop

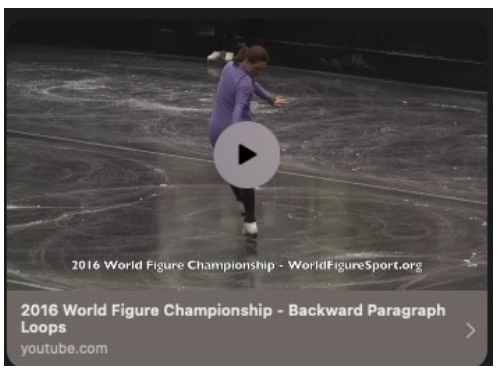


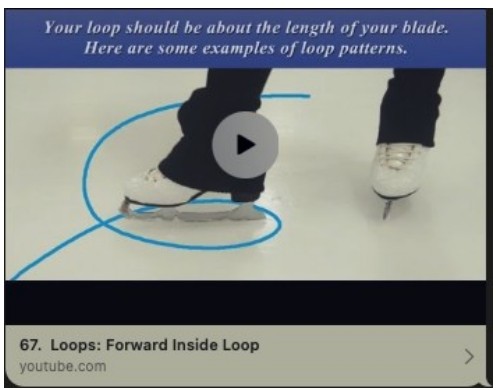
The Scribe-a-Loop was used to create loop tracings on the ice for training. It is considered to be the “perfect” loop shape by its maker and most others skaters. Thought to be the only one remaining in the world, its maker is unknown but “A. KOCH” is stamped on the back. Likely 20th Century.

Donated to World Figure Sport by Barbara Bennett

Watch Loop Videos

- 2016 World Figure Championship – Change Loops
- 2016 World Figure Championship – Backward Paragraph Loops
- Kseniya and Oleg: Forward Inside Loop





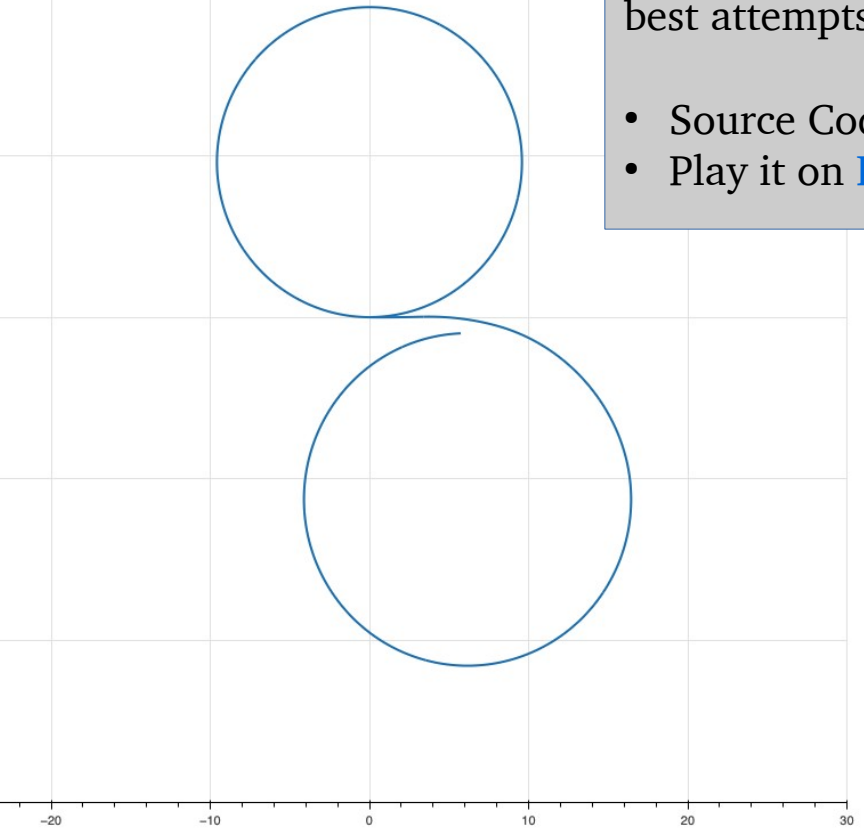
Conclusions

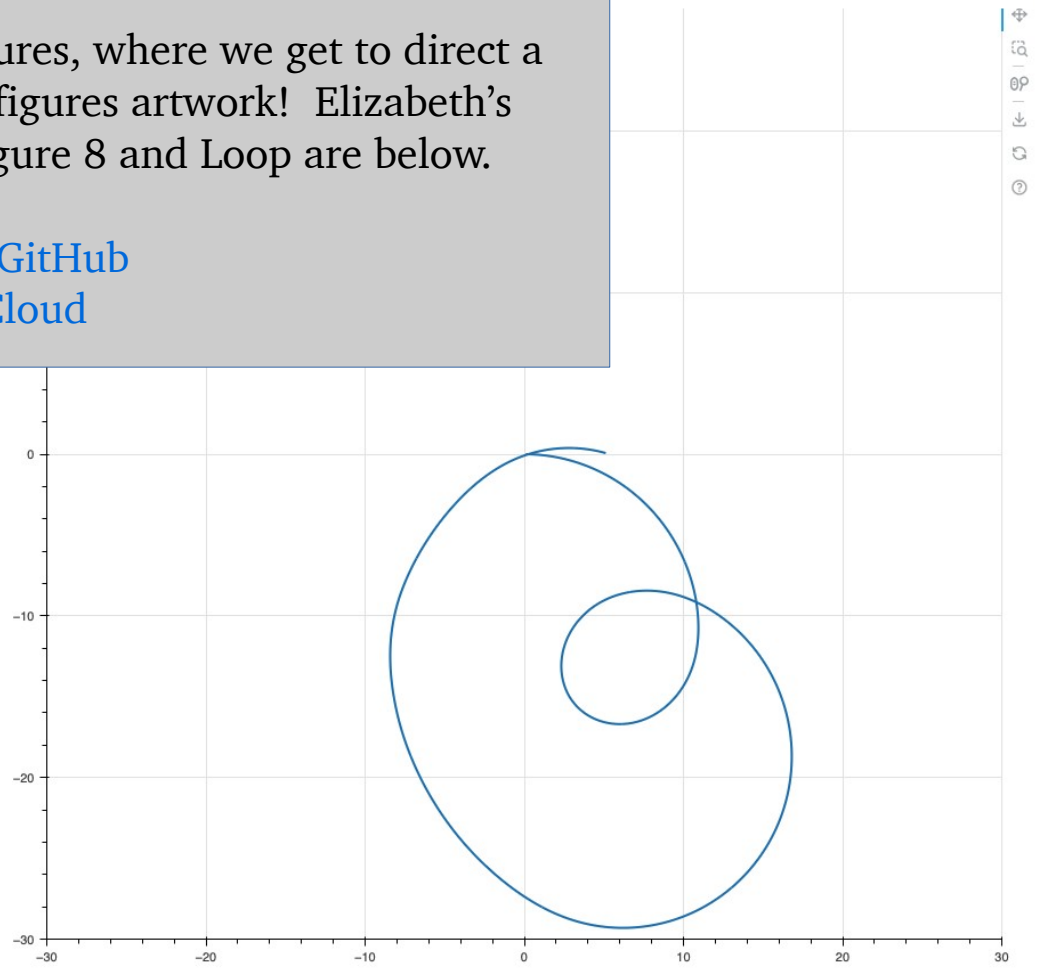
- Clothoid shape is the way to execute a figure skating loop most smoothly on entry and exit.
- Figures skaters learned this intuitively in their bodies in first half of 19th Century.
- Skaters were just one of many people repeatedly re-discovering the Euler Spiral.
- The link between Euler Spiral and Figure Skating was not made until 2025 (Fischer).
- *https://artofskating.org*

Play YP Figures!

Welcome to YP Figures, where we get to direct a Yellow Pig skating figures artwork! Elizabeth's best attempts at Figure 8 and Loop are below.

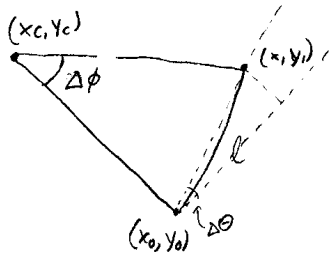
- Source Code on GitHub
- Play it on Posit Cloud





YP Figures Formulas

These formulas were used to develop the YP Figures simulation game. A first-order explicit integration in time is shown.



V = speed (scalar)

Δt = timestep

θ = direction of travel

$\theta' = \frac{d\theta}{dt}$ = Rate of turn (steering wheel position)

r = Radius of curvature to travel arc of length $\ell = V\Delta t$ while changing angle of $\Delta\theta = \theta'\Delta t$

$$\ell = r\,\Delta\theta = r\theta'\Delta t = V\Delta t$$
$$\Rightarrow r = V/\theta'$$

(x_0, y_0) = Position @ t_0

(x_1, y_1) = " " t_1

(x_c, y_c) = Center of circle of radius r

ϕ = Absolute position around the circle $= \theta + \frac{\pi}{2}$

$$\Delta\theta = \theta'\Delta t = \Delta\phi \qquad \theta_1 = \theta_0 + \theta'\Delta t$$

$$x_1 - x_0 = r\left[\cos\left(\theta_1 + \tfrac{\pi}{2}\right) - \cos\left(\theta_0 + \tfrac{\pi}{2}\right)\right]$$
$$= \tfrac{V}{\theta'}\left[\cos\left(\theta_0 + \theta'\Delta t + \tfrac{\pi}{2}\right) - \cos\left(\theta_0 + \tfrac{\pi}{2}\right)\right]$$

$$\theta_1 - \theta_0 = \theta'\Delta t$$